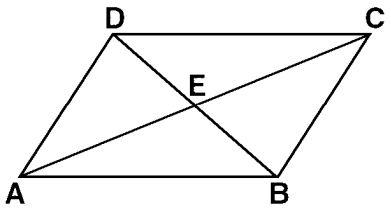
Name: _____

CC Geometry Honors

Proofs with Parallelograms

Questions 1 through 4 refer to the following:

Given: Quadrilateral *ABCD* below



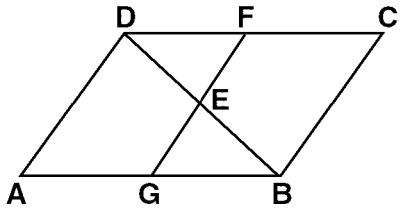
1) If $\overline{AB} \parallel \overline{CD}$ and $\overline{AD} \cong \overline{BC}$, determine whether quadrilateral *ABCD* is a parallelogram. [*Explain your answer.*]

2) If $AE = EC$ and $DE = EB$, determine whether quadrilateral *ABCD* is a parallelogram. [*Explain your answer.*]

3) If $\overline{AD} \parallel \overline{BC}$ and $\overline{AD} \cong \overline{BC}$, determine whether quadrilateral *ABCD* is a parallelogram. [*Explain your answer.*]

4) If $\overline{AD} \cong \overline{DC}$ and $\overline{AB} \cong \overline{BC}$, determine whether quadrilateral *ABCD* is a parallelogram. [*Explain your answer.*]

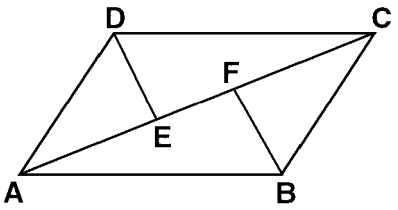
5)



Given: *ABCD* is a parallelogram
$\overline{FG}$ bisects $\overline{DB}$

Prove: $\overline{FE} \cong \overline{EG}$

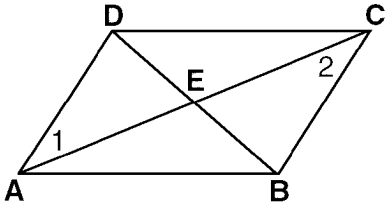
6)



Given: $ABCD$ is a parallelogram
$\overline{DE} \perp \overline{AC}$
$\overline{BF} \perp \overline{AC}$

Prove: $\overline{AE} \cong \overline{FC}$

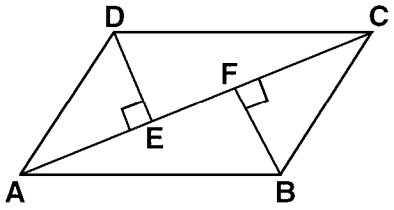
7)



Given: $\overline{DB}$ bisects $\overline{AC}$
$\angle 1 \cong \angle 2$

Prove: $ABCD$ is a parallelogram

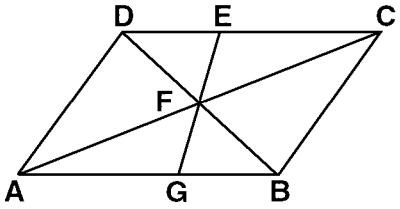
8)



Given: $\overline{DE} \perp \overline{AC}$
$\overline{BF} \perp \overline{AC}$
$\overline{AE} \cong \overline{FC}$
$\overline{DE} \cong \overline{FB}$

Prove: *ABCD* is a parallelogram

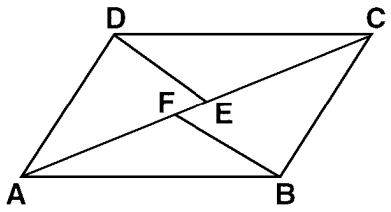
9)



Given: *ABCD* is a parallelogram
$\overline{AC}$, $\overline{BD}$, and $\overline{GE}$ intersect at *F*

Prove: $\overline{EF} \cong \overline{FG}$

10)



Given: Parallelogram $ABCD$
$\overline{AE} \cong \overline{FC}$

Prove: $\triangle DEC \cong \triangle BFA$

1) No
SAMPLE EXPLANATION: The same pair of sides must be parallel and congruent.

2) Yes
SAMPLE EXPLANATION: If the diagonals of a quadrilateral bisect each other, the quadrilateral is a parallelogram.

3) Yes
SAMPLE EXPLANATION: If 2 sides of a quadrilateral are parallel and congruent, the quadrilateral is a parallelogram.

4) No
SAMPLE EXPLANATION: The opposite sides must be congruent.

5) SAMPLE PROOF:
(1) $ABCD$ is a parallelogram, $\overline{FG}$ bisects $\overline{DB}$ (Given)
(2) $\overline{DC} \parallel \overline{AB}$ (Opposite sides of a parallelogram are parallel.)
(3) $\angle CDE \cong \angle ABE$ (If two parallel lines are cut by a transversal, the alternate interior angles are congruent.)
(4) $\angle DEF \cong \angle BEG$ (If two lines intersect, the vertical angles are congruent.)
(5) $\overline{DB} \cong \overline{EB}$ (The bisector of a segment is a point, line or plane that divides the segment into two congruent segments.)
(6) $\triangle DEF \cong \triangle BEG$ ($ASA \cong ASA$)
(7) $\overline{FE} \cong \overline{EG}$ (CPCTC)

6) SAMPLE PROOF:
(1) $ABCD$ is a parallelogram, $\overline{DE} \perp \overline{AC}$, $\overline{BF} \perp \overline{AC}$ (Given)
(2) $\angle DEA \cong \angle BFC$ (Perpendicular lines form congruent right angles.)
(3) $\overline{DA} \parallel \overline{BC}$ (Opposite sides of a parallelogram are parallel.)
(4) $\angle DAE \cong \angle BCF$ (If two parallel lines are cut by a transversal, the alternate interior angles are congruent.)
(5) $\overline{DA} \cong \overline{BC}$ (Opposite sides of a parallelogram are congruent.)
(6) $\triangle ADE \cong \triangle CBF$ ($AAS \cong AAS$)
(7) $\overline{AE} \cong \overline{FC}$ (CPCTC)

7) SAMPLE PROOF:
(1) $\overline{DB}$ bisects $\overline{AC}$, $\angle 1 \cong \angle 2$ (Given)
(2) $\overline{AE} \cong \overline{CE}$ (The bisector of a segment is a point, line or plane that divides the segment into two congruent segments.)
(3) $\angle DEA \cong \angle BEC$ (If two lines intersect, the vertical angles are congruent.)
(4) $\triangle AED \cong \triangle CEB$ ($ASA \cong ASA$)
(5) $\overline{AD} \cong \overline{CB}$ (CPCTC)
(6) $\overline{AD} \parallel \overline{CB}$ (If two lines are cut by a transversal, so that the alternate interior angles are congruent, the lines are parallel.)
(7) $ABCD$ is a parallelogram (If a quadrilateral has one pair of sides both parallel and congruent, the quadrilateral is a parallelogram.)

8) SAMPLE PROOF:
(1) $\overline{DE} \perp \overline{AC}$, $\overline{BF} \perp \overline{AC}$, $\overline{AE} \cong \overline{FC}$, $\overline{DE} \cong \overline{FB}$ (Given)
(2) $\angle DEA \cong \angle BFC$ (Perpendicular lines form congruent right angles.)
(3) $\triangle DEA \cong \triangle BFC$ ($SAS \cong SAS$)
(4) $\overline{DA} \cong \overline{BC}$, $\angle DAE \cong \angle BCF$ (CPCTC)
(5) $\overline{DA} \parallel \overline{BC}$ (If two lines are cut by a transversal, so that the alternate interior angles are congruent, the lines are parallel.)
(6) $ABCD$ is a parallelogram (If a quadrilateral has one pair of sides both parallel and congruent, the quadrilateral is a parallelogram.)

9) SAMPLE PROOF:
(1) $ABCD$ is a parallelogram., $\overline{AC}$, $\overline{BD}$, and $\overline{GE}$ intersect at F (Given)
(2) $\overline{DF} \cong \overline{BF}$ (The diagonals of a parallelogram bisect each other.)
(3) $\overline{DC} \parallel \overline{AB}$ (Opposite sides of a parallelogram are parallel.)

(4) $\angle BDC \cong \angle ABD$ (If two parallel lines are cut by a transversal, the alternate interior angles are congruent.)
(5) $\angle DFE \cong \angle BFG$ (If two lines intersect, the vertical angles are congruent.)
(6) $\triangle DFE \cong \triangle BFG$ ($ASA \cong ASA$)
(7) $\overline{EF} \cong \overline{FG}$ (CPCTC)

10) SAMPLE *SAS* PROOF:
(1) Parallelogram $ABCD$, $\overline{AE} \cong \overline{FC}$ (Given)
(2) $\overline{DC} \cong \overline{AB}$ (Opposite sides of a parallelogram are congruent.)
(3) $\overline{DC} \parallel \overline{AB}$ (Opposite sides of a parallelogram are parallel.)
(4) $\angle BAF \cong \angle DCE$ (If two parallel lines are cut by a transversal, the alternate interior angles are congruent.)
(5) $\overline{FE} \cong \overline{FE}$ (Reflexive Property of Equality)
(6) $AE - FE = FC - FE$, $\overline{AF} \cong \overline{CE}$ (Subtraction Property of Equality)
(7) $\triangle DEC \cong \triangle BFA$ ($SAS \cong SAS$)